

# Hands-on: Line radiative transfer

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January 31, 2023

## 1 Installation

### 1.1 Fortran compiler

You will work on your laptop. You need a Fortran compiler available (e.g. gfortran). Check this is the case:

```
$ which gfortran
/usr/bin/gfortran
```

### 1.2 Code Installation

Download the code at URL:

[http://www.sron.rug.nl/~vdtak/radex/radex\\_public.tar.gz](http://www.sron.rug.nl/~vdtak/radex/radex_public.tar.gz)

Then proceed with the installation of RADEX. Please consult this URL:

<http://www.sron.rug.nl/~vdtak/radex/index.shtml>

and follow the steps to install the program on your machine.

### 1.3 Input data files

To perform a RADEX calculation, you need fundamental atomic or molecular data. These include: energy levels, transition frequencies, Einstein coefficient  $A_{ul}$  for each transition, collision coefficient  $C_{ul}$  for each transition. Tabulated values are available in various publications and databases. You will use the EMAA database at URL:

<https://emaa.osug.fr/>

Search for the following species: CO and CS. In each case, select only para-H<sub>2</sub> as collision partner. Finally, move the EMAA datasets to the Radex/data/ directory.

### 1.4 Doing plots

If using gnuplot, you will need data files organized in columns. The RADEX output file will look like:

```
$ cat example.out
* Radex version      : 30nov11
* Geometry           : Uniform sphere
* Molecular data file : data/hco+.dat
* T(kin)              [K]: 20.000
* Density of H2       [cm-3]: 1.000E+04
```

```

* T(background)      [K]:      2.730
* Column density [cm-2]: 1.000E+13
* Line width        [km/s]:      1.000
Calculation finished in 28 iterations

```

LINE	E_UP (K)	FREQ (GHz)	WAVEL (um)	T_EX (K)	TAU	...
1 -- 0	4.3	89.1884	3361.3393	4.505	4.686E+00	...
2 -- 1	12.8	178.3748	1680.6883	3.769	5.300E+00	...
3 -- 2	25.7	267.5573	1120.4795	3.724	8.856E-01	...
4 -- 3	42.8	356.7338	840.3814	6.033	3.652E-02	...
5 -- 4	64.2	445.9024	672.3275	9.702	2.527E-03	...

To plot the  $T_{ex}$  as a function of frequency, you may type:

```

gnuplot>set xlabel "Frequency (GHz)"
gnuplot>set ylabel "Tex (K)"
gnuplot>plot 'example.out' u 5:7 w l title "Tex"

```

## 2 Practical work

In all the calculations involving RADEX, you will choose the frequency range such that it contains only a few rotational transitions ( $< 10$ ). This range will depend on the molecule considered. The only background radiation field will be the cosmic microwave background (CMB) at 2.73 K. You will store all your calculations in ASCII files, in order to make plots.

### 2.1 Preliminary calculations

1. Get the dipole moments  $\mu$  of the CO and CS molecules from the CDMS catalog at URL: <https://cdms.astro.uni-koeln.de/classic/entries/>.
2. Extract the  $A_{ul}$  and  $C_{ul}$  values from the EMAA datasets, and compute the critical density  $n_{cr}$  of the  $j = 1 \rightarrow 0$  and  $j = 2 \rightarrow 1$  transitions of CO and CS molecules for a kinetic temperature of 10 K. Verify that they approximately scale as  $n_{cr} \propto \nu^3 \mu^2$ .
3. Determine the rotational constants of CS and CO from their ground state transition frequencies, and compare with the tabulated values from the CDMS.
4. Plot the ('high-temperature') linear approximation of the partition function for CO and compare to the CDMS values. You may also want to compute it directly from degeneracies and level energies.

### 2.2 Verify equilibrium limits

1. How can you drive the molecular populations towards LTE ? And towards radiative equilibrium with the CMB ?
2. Choose a set of parameters that fulfill the LTE approximation for some rotational transitions. Check with RADEX that the chosen transition is thermalized. Idem for radiative equilibrium with the CMB.
3. Compare the calculated populations of CO with the one corresponding to LTE.
4. Using the radiative transfer solution and the  $T_{ex}$  and opacity calculated by RADEX, compute the radiation temperature and check against the value given by RADEX.

### 2.3 Transition from sub-thermal to thermalized transition

Choose one species, and at a given kinetic temperature:

1. Compute with RADEX the excitation temperature of the selected rotational transitions, for densities of the collider (para-H<sub>2</sub>) in a range large enough to encompass the critical density of each transition.
2. Trace the excitation temperature of each transition, as a function of density.
3. Interpret the result. In particular, what transitions thermalize first.

### 2.4 Photon trapping

Choose one species, and one kinetic temperature.

1. For a given column density per unit velocity interval  $N/\Delta v$ , compute the opacity of the selected rotational transitions. Repeat the same calculations for a  $N/\Delta v$  ten times larger. How does the opacity change ?
2. Find a value of  $N/\Delta v$  for which the opacities are small ( $< 1$ ) and compute the excitation temperature for a set of densities encompassing the critical densities of the transitions considered. Repeat the same calculations for much larger values of  $N/\Delta v$  (so that all opacities are larger than 1). Plot again the  $T_{ex}$  as functions of the density. What is the effect of the opacity on thermalization. Can you explain this result formally ?

### 2.5 Choosing the right transition

We want to determine the physical conditions with some precisions, in gas for which we have a mere idea of its physical conditions. The question is thus which species and which transition to choose. You will use  $N(\text{CO})=10^{15} \text{ cm}^{-2}/(\text{km/s})$  and  $N(\text{CS})=10^{13} \text{ cm}^{-2}/(\text{km/s})$  to limit opacity effects. Choose temperatures lower than 40 K.

1. Run RADEX at constant  $T_{kin}$  and varying densities. Plot the radiation temperatures ( $T_R$ ) of the selected transitions against the density. Repeat the calculations for various kinetic temperatures, and compare the results.
2. Same as above, but now plotting the lower level populations. Conclude.
3. What is the best tracer of density when  $n(\text{H}_2) > 10^5 \text{ cm}^{-3}$ : CO or CS ?